90% Confidence limit for $\mu_{\nu\tau}$ using the Feldman-Cousins method

Reinhard Schwienhorst
University of Minnesota

Short report
E872 phone meeting, 10/19/99

Outline

- Introduction
- Statistics
  - Bayesian and Frequentist intervals
- Feldman-Cousins method
  - Likelihood ratio
  - Confidence belt
- Conclusions
- Outlook

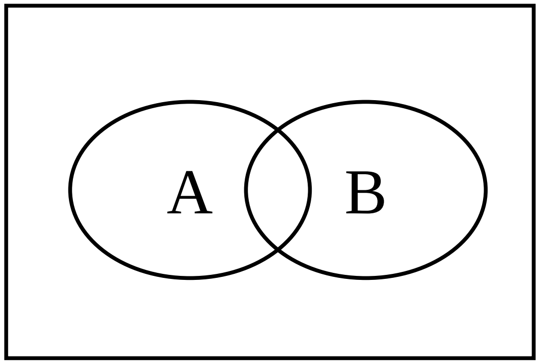
Introduction

- At the August meeting I presented a preliminary result for μ_{ν_τ}
  - 90% confidence limit
  - simple classical (frequentist) statistics
  - not appropriate for low statistics + background
- Many Physicists use “Bayesian” statistics
  - works for low statistics+background
  - requires knowledge of prior probability distribution function (pdf) of the parameter to estimate
- Feldman and Cousins have specified a method to find correct “frequentist” confidence intervals for low statistics + background
  - used in neutrino oscillation searches

Statistics

- Probability of observing a value x if the true value is μ_t: $P(x|\mu_t)$
- Classical statistics relies only on $P(x|\mu_t)$
  - statements about x

- Bayes theorem for two sets A and B: $P(A|B)\,P(B) = P(B|A)\,P(A)$
- applied to measurements: $P(\mu_t|\,x_0) = L(x_0|\mu_t)\,P(\mu_t)\,/\,P(x_0)$
  - result of a specific experiment: x_0
  - $L\,(x_0|\mu_t)$: likelihood function, $=P(x_0|\mu_t)$
  - $P\,(\mu_t)$: prior pdf, needs to be specified
- This is Bayesian statistics
  - statements about μ_t



Bayesian and Frequentist statements

- Statement about the value x_0 observed in a single measurement and the true value μ_t and a 90% confidence interval
- Frequentist (classical):
  If I repeat the experiment many times (and create many confidence intervals), the true value μ_t will lie inside the classical interval 90% of the time.
- Bayesian:
  If I observe x_0 in a single experiment, 90% of the possible values for μ_t lie inside the Bayesian interval.

Feldman-Cousins method

- Problem region: small number of observed events with background
- Frequentist method
- ordering principle to treat the low statistics region properly
  - order based on likelihood ratio
  - increase the interval until the probability sum is $\geq 90\%$

Likelihood ratio

- For each possible signal mean μ_s, calculate the possible number of events

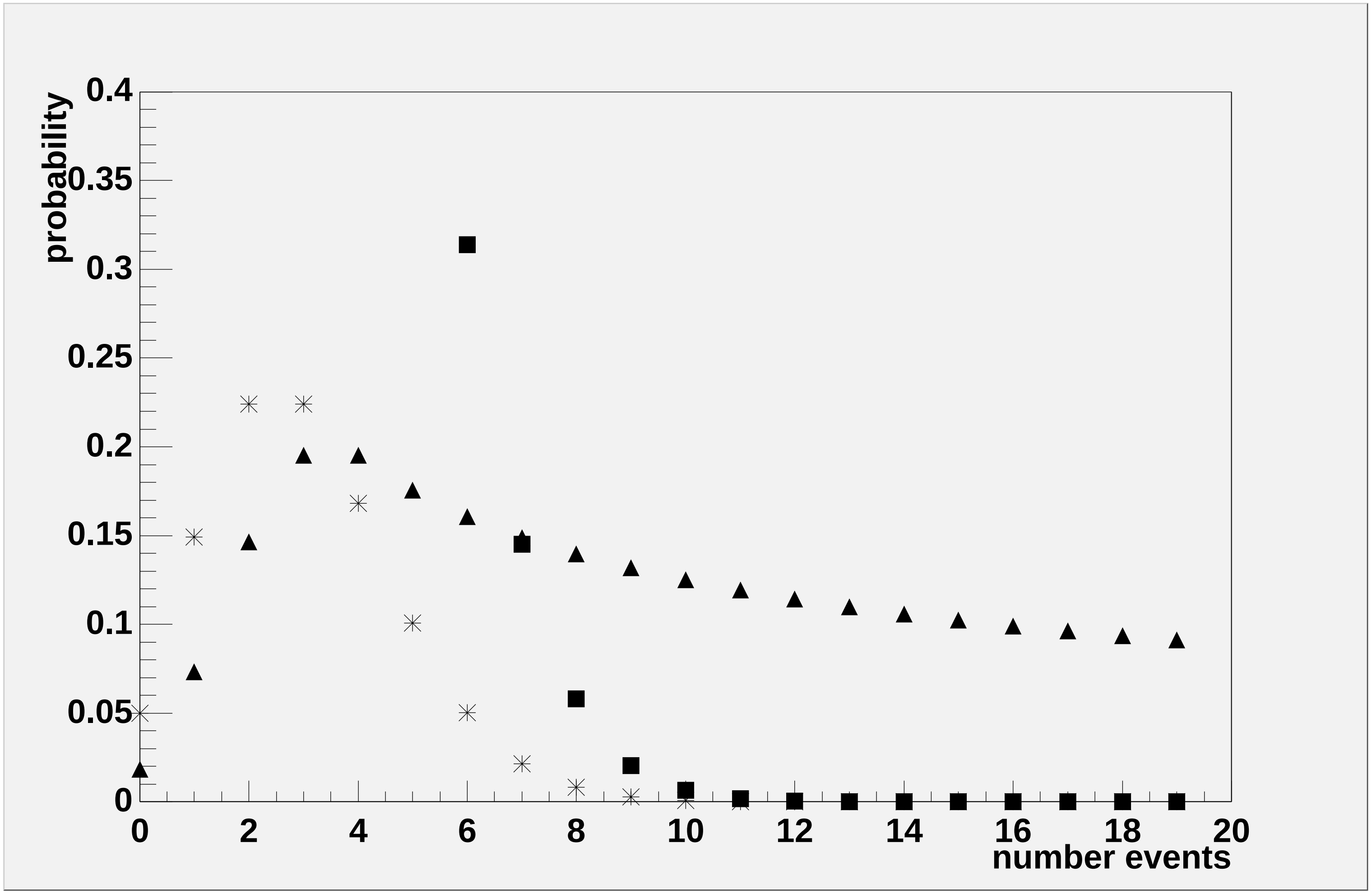
  $n_{sum} = n_s + n_{bg}$ (signal + background)
- Also calculate the probability of the "best" distribution
  - $\mu_{best} = n_{sum} - n_{bg}$ (and require $\mu_{best} \geq 0$)
- The likelihood ratio is

$$R = \frac{P(\mathrm{n}_{\mathrm{sum}}, \mu_s)}{P(\mathrm{n}_{\mathrm{sum}}, \mu_{\mathrm{best}})}$$

- Extend the n_s interval in decreasing *R* order until

$$\sum_{n_s} P(n_{\mathrm{sum}}, \mu_s) > 0.9$$

Poisson distribution and ratio R



* : Poisson distribution, μ=3

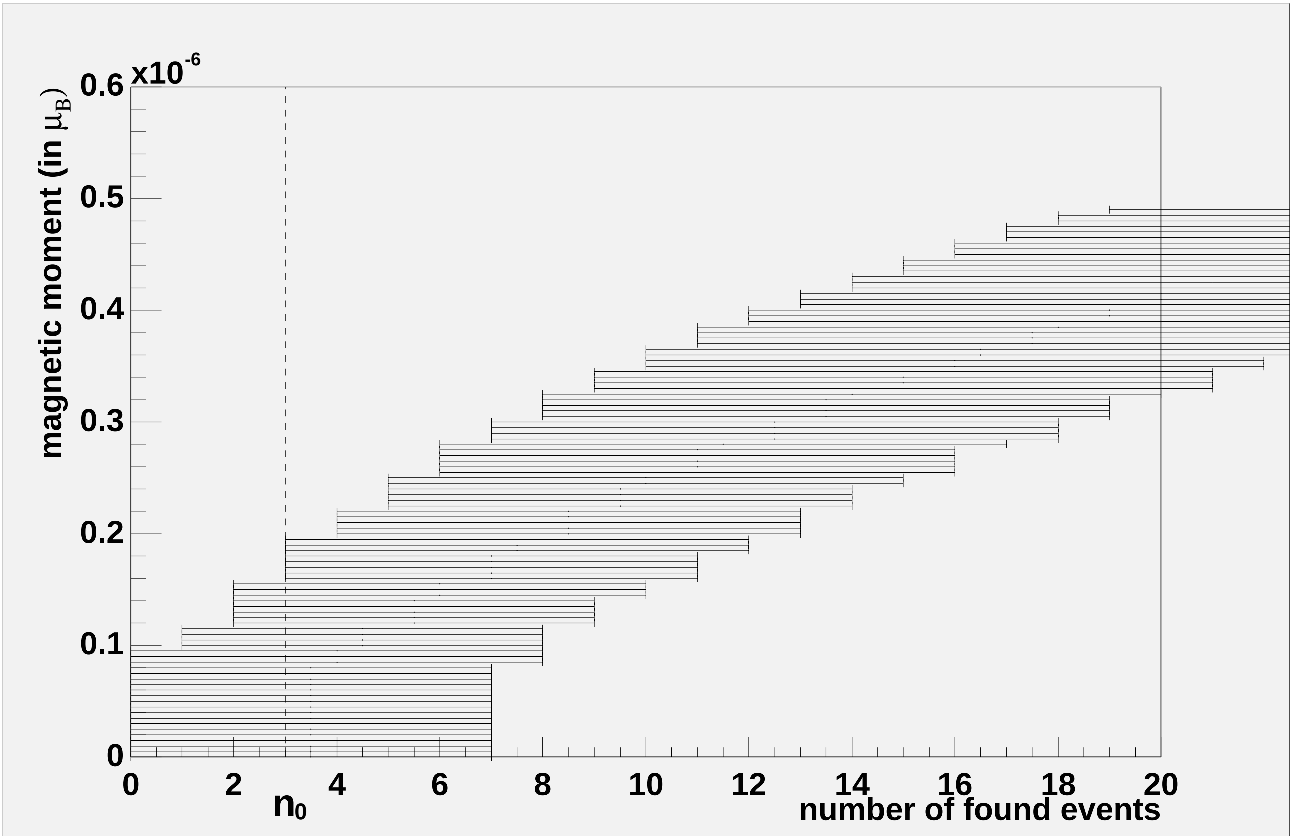
▲ : distribution of the “best” value

■ : ratio R

Confidence belt

- Calculate an acceptance interval for many values μ_s
- Make a graph of the acceptance intervals versus μ_s
- example: magnetic moment μ_{ν_τ}
  - background n_{bg}=4 events,
  - observed n_0=3 events
  - for each possible value μ_{ν_τ}, create a 90% acceptance interval of observed events

Confidence belt



- A horizontal line corresponds to a 90% acceptance interval
  - actually ≥90% due to discreteness
  - created without prior knowledge of the result
- A vertical line corresponds to a 90% confidence interval
  - 90% confidence limit for μ_{ν_τ} : $2\times10^{-7}\mu_B$

Conclusions

- The Feldman-Cousins method is widely used in neutrino physics
  - Frequentist approach
  - It deals properly with small numbers
  - It does not yet allow for uncertainty in flux or background
    - usually the statistical error dominates
    - systematic error can safely be ignored up to $\approx$30%

Outlook

- I will use the Feldman-Cousins method to compute a 90% upper confidence limit on μ_{ν_τ}
- I will also quote the observed number of events and the expected number of background events